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The efficiency of a turbine in a tidal channel

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There is an upper bound to the amount of power that can be generated by turbines in tidal channels as too many turbines merely block the flow. One condition for achievement of the upper bound is that the turbines are deployed uniformly across the channel, with all the flow through them, but this may interfere with other uses of the channel. An isolated turbine is more effective in a channel than in an unbounded flow, but the current downstream is non-uniform between the wake of the turbines and the free stream. Hence some energy is lost when these streams merge, as may occur in a long channel. We show here, for ideal turbine models, that the fractional power loss increases from 1/3 to 2/3 as the fraction of the channel cross-section spanned by the turbines increases from 0 to close to 1. In another scenario, possibly appropriate for a short channel, the speed of the free stream outside the turbine wake is controlled by separation at the channel exit. In this case, the maximum power obtainable is slightly less than proportional to the fraction of the channel cross-section occupied by turbines.

1. Introduction

The generation of electricity by turbines in strong tidal currents is analogous to the use of wind turbines if the cross-sectional area of the turbine is small compared with that of the channel. A classic result is that the maximum available power is $\frac{8}{27}\rho A u^3$, where ρ is the fluid density, A is the cross-sectional area of the turbine and u the upstream flow speed. This power is 16/27 or 59 % of the upstream kinetic energy flux through an area equal to the turbine cross-section, with the fraction often, though perhaps misleadingly, referred to as the efficiency of the turbine. This result is commonly attributed to Betz (1920, 1926), though, as discussed by Bergey (1980), it was clearly derived earlier by Lanchester (1915) in his Problem II. As described by Lanchester (1915), the results for the equivalent case of a propeller were even anticipated by Froude. (Lanchester cites Froude's work with the memorable comments: "It is, as I understand it, quite candidly admitted by Mr. Froude that the régime contemplated by his theory is not capable of exact expression. This, however, is no obstacle to the application of any theory in real hydrodynamics; if it were necessary for the engineer to await the pleasure of the pure mathematician in these matters, the subject would have made scarcely perceptible progress since the time of Noah.") We shall refer to the limiting case of maximum power as the Lanchester–Betz limit. In it, the flow speed decreases to $\frac{2}{3}u$ through the turbine and further to $\frac{1}{3}u$ as the flow expands in the turbine wake.

For tidal channels, where one might want to deploy arrays of turbines, the channel boundaries are likely to have an effect, but the kinetic energy flux through the channel is still frequently used as a metric of the power available. There is no theoretical basis for this and an immediate difficulty is that this component of the energy flux is not a conserved quantity in a channel of varying cross-section. Moreover, as more turbines are added, the current will eventually slow and the power decrease. Garrett & Cummins (2005, henceforth referred to as GC05), examined the problem for a tidal channel connecting two large bodies of water in which the tides were assumed to be unaffected by changes in the connecting channel. They assumed that the volume flux through the channel is only a function of time (as for a channel which is short compared with a tidal wavelength), and that the cross-sectional area at any location does not change significantly with time (as for flow at low Froude number and with a tidal range which is not a significant fraction of the water depth). They further assumed that arrays, or 'fences', of tidal turbines occupy the whole cross-section with all the water flow through them, and that the tidal current is uniform across the channel. (The assumption of a complete tidal fence is equivalent to assuming that the channel cross-section is occupied by a single turbine, though clearly this is unlikely on geometrical grounds.) Finally, this approach neglects the effect of support structures and the associated loss of head as turbine wakes merge with the stagnant water in the lee of the supports.

The assumptions of GC05 permit integration along the channel of the partial differential equation (with respect to the along-channel coordinate x and time t) for the current speed and lead to an ordinary differential equation for the volume flux $Q(t)$ along the channel. The turbine fences are represented by a frictional force F in addition to the natural one and the effect of flow separation at the exit from the channel. The power generated (disregarding internal turbine inefficiency) is just the time average of the force F and volume flux Q. GC05 showed that, as expected, the power at first increases as more turbines are added but then decreases as the flow is choked. They found that the maximum power can be expressed as $\gamma \rho g a Q_{\max}$ where ρ is the water density, g is acceleration due to gravity, a is the amplitude of the assumed sinusoidal sea level difference between the two ends of the channel, $Q_{\max}$ is the maximum volume flux through the channel in the natural state, and γ is a coefficient which varies over the narrow range between 0.24 and 0.20 as the natural flow regime varies from being dominated by acceleration to being dominated by friction and flow separation at the channel exit. The results of this simple model have been supported by a detailed numerical model for a particular tidal channel (Sutherland, Foreman & Garrett 2007).

In practice, the use of a complete tidal fence occupying the whole cross-section of a channel is likely to be ruled out by the needs of shipping, fish and marine mammals. It is still likely, however, that partial fences will occupy a significant fraction of the cross-section. The purpose of this paper is thus to extend earlier studies by allowing for turbines which occupy less than the whole cross-section of a channel, but are nonetheless affected by the lateral boundaries.

2. Single turbine in a channel

Consider a single turbine of cross-sectional area A in a channel of uniform cross-sectional area A_c (figure 1). (For open-channel flow in a uniform channel with a free surface, taking A_c to be constant is valid if the Froude number of the flow is small. This is typically the case in tidal flows.) The streamtube for steady flow through the turbine has a cross-sectional area A_0 far upstream in undisturbed flow where the pressure p_0 and the flow speed u_0 are assumed uniform across the channel. (Here

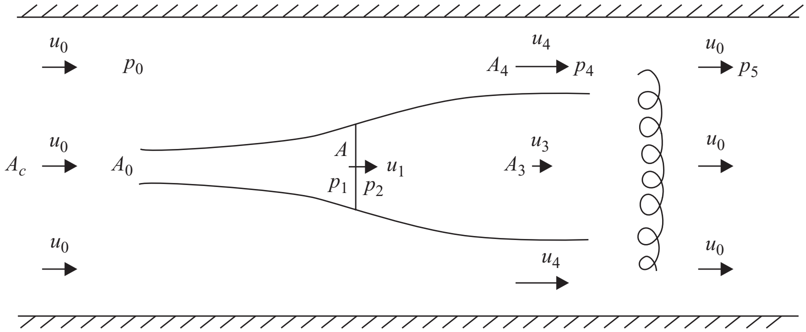

FIGURE 1. Definition sketch for a single turbine in a channel.

we use the term 'pressure' to describe the true pressure $+\rho g z$, with z the vertical coordinate, positive upwards.)

The pressure in the same streamtube is p_1 just upstream of the turbine, and p_2 just downstream, both assumed uniform across the turbine, as in the Lanchester–Betz formulation. The flow speed u_1 through the turbine is also taken to be uniform. As illustrated in figure 1, the streamtube continues to expand in the wake downstream of the turbine before settling to a constant area A_3 with speed u_3. Outside the wake, the flow speed is u_4 and the pressure across the whole channel is p_4. Any swirling component of the flow is ignored, as in the Lanchester–Betz analysis which would have $A_c \to \infty$, $p_4 = p_0$, and $u_4 = u_0$. We allow for a further downstream evolution of the flow, by lateral mixing, to an eventual return to a laterally uniform speed u_0, with a pressure p_5 which is now different from the upstream pressure p_0.

Our analysis will be based on combining integral forms of the continuity, momentum, and energy equations. We start with the continuity equations for the streamtube and the whole channel. They are, respectively,

$$A_0 u_0 = A u_1 = A_3 u_3, \tag{2.1}$$

$$(A_c - A_3) u_4 + A_3 u_3 = A_c u_0, \tag{2.2}$$

and (2.2) may be written as

$$A_c(u_4 - u_0) = A_3(u_4 - u_3). \tag{2.3}$$

Next we consider the momentum budget for the water between the upstream section and the downstream section in the wake before it has mixed laterally. This gives the force F on the turbine as

$$F = A_c u_0^2 - (A_c - A_3) u_4^2 - A_3 u_3^2 + (p_0 - p_4) A_c \tag{2.4}$$

where we have taken the density $\rho = 1$ to avoid carrying it through the formulae. Now Bernoulli's equation along a streamline outside the streamtube containing the turbine implies that

$$p_0 - p_4 = \tfrac{1}{2}\left(u_4^2 - u_0^2\right) \tag{2.5}$$

so that, using also (2.3),

$$F = \tfrac{1}{2} A_3 (u_4 - u_3)(u_4 + 2u_3 - u_0). \tag{2.6}$$

The force F may also be derived using Bernoulli's equation from upstream to the turbine and again from the turbine to the wake section. This, combined with (2.5), gives

$$p_1 - p_2 = \tfrac{1}{2}\left(u_4^2 - u_3^2\right). \tag{2.7}$$

The force on the turbine is just its area times this pressure difference, so that

$$F = \tfrac{1}{2}A\left(u_4^2 - u_3^2\right). \tag{2.8}$$

For (2.6) and (2.8) to agree, we require

$$u_1 = \frac{u_3(u_4 + u_3)}{u_4 + 2u_3 - u_0}, \tag{2.9}$$

where use has been made of (2.1).

The power generated by the turbine, assuming no internal losses, is $P = Fu_1$ or, using (2.8) and (2.9),

$$P = \tfrac{1}{2}A\frac{u_3(u_4 + u_3)\left(u_4^2 - u_3^2\right)}{u_4 + 2u_3 - u_0}. \tag{2.10}$$

This result is consistent with evaluation of the energy budget for the same control volume used for (2.4), giving

$$P = A_c\left(p_0u_0 + \tfrac{1}{2}u_0^3\right) - (A_c - A_3)\left(p_4u_4 + \tfrac{1}{2}u_4^3\right) - A_3\left(p_4u_3 + \tfrac{1}{2}u_3^3\right), \tag{2.11}$$

and then using the continuity equation (2.3) and the Bernoulli equation (2.5) above.

As $A_c \to \infty$ and consequently $u_4 \to u_0$, (2.9) becomes the Lanchester–Betz result

$$u_1 = \tfrac{1}{2}(u_0 + u_3), \tag{2.12}$$

and (2.10) reduces to

$$P = P_{\text{LB}} = \tfrac{1}{4}A(u_0 + u_3)\left(u_0^2 - u_3^2\right) = \tfrac{1}{2}(1 + r)(1 - r^2) \times \tfrac{1}{2}Au_0^3 \tag{2.13}$$

where $r = u_3/u_0$ and $\frac{1}{2}Au_0^3$ is the energy flux of the undisturbed flow. The right-hand side of (2.13) has a maximum at $r = 1/3$ of $16/27$, or 0.59, the classic Lanchester–Betz result. One objective of the present study is to see how this changes if the ideal turbine is confined within a channel.

We also seek to compare the energy obtained from the isolated turbine with the power that would be obtained if the flow were to pass through a turbine fence occupying the whole cross-section of the channel. This reference power is

$$P_{\text{ref}} = A_cu_0(p_0 - p_5) \tag{2.14}$$

where p_5 is the pressure downstream at a location where the slow-moving wake has merged with the faster surrounding flow. As stressed by Garrett & Cummins (2004), energy is lost in this merging so that P must be less than P_{ref}. We obtain p_5 by evaluating the consequences of momentum flux conservation between the wake section and this new section farther downstream. This gives

$$(A_c - A_3)\left(p_4 + u_4^2\right) + A_3\left(p_4 + u_3^2\right) = A_c\left(p_5 + u_0^2\right). \tag{2.15}$$

Using (2.3) to express A_3 in terms of A_c and (2.5) to replace p_4 with p_0, we obtain

$$p_0 - p_5 = \tfrac{1}{2}(u_4 - u_0)(u_4 + 2u_3 - u_0) \tag{2.16}$$

so that

$$P_{\text{ref}} = A_cu_0\tfrac{1}{2}(u_4 - u_0)(u_4 + 2u_3 - u_0). \tag{2.17}$$

Using (2.3) and (2.6), this is simply

$$P_{\text{ref}} = Fu_0, \tag{2.18}$$

and

$$\frac{P}{P_{\text{ref}}} = \frac{u_1}{u_0}. \tag{2.19}$$

As pointed out by Corten (2001) for a turbine in an unbounded flow, (2.18) is readily obtained by considering the problem in a frame of reference in which the turbine moves upstream at speed u_0 into fluid at rest; the rate of working Fu_0 then supplies the energy of the differential flow in the wake as well as the power Fu_1 extracted by the turbine. In this frame of reference the wake energy is lost as the wake merges with its surroundings.

We also note from (2.5) and (2.16) that

$$p_5 - p_4 = (u_4 - u_0)(u_0 - u_3). \tag{2.20}$$

Now $u_4 > u_0$ and $u_3 < u_0$, so that $p_5 > p_4$ even though energy is dissipated between the sections with pressures p_4 and p_5.

We now need to express P from (2.10) and P_{ref} from (2.17) or (2.19) in terms of A, A_c, u_0, and u_3. We can obtain u_1 from (2.9), and so the immediate problem is to solve for u_4. Starting from (2.1) and (2.2), we have

$$u_3(u_4 - u_0) = \epsilon u_1(u_4 - u_3) \quad \text{where} \quad \epsilon = A/A_c. \tag{2.21}$$

Using (2.9) for u_1, we obtain

$$(1-\epsilon)u_4^2 - 2(u_0 - u_3)u_4 + u_0^2 - 2u_0u_3 + \epsilon u_3^2 = 0 \tag{2.22}$$

whence

$$u_4 = \frac{u_0 - u_3 + \left[\epsilon u_0^2 - 2\epsilon u_0 u_3 + (1 - \epsilon + \epsilon^2)u_3^2\right]^{1/2}}{1-\epsilon}. \tag{2.23}$$

(The positive root is necessary, as is readily seen in the limit for small ϵ.) We emphasize that (2.23) is a simple consequence of continuity and momentum conservation; it is equivalent to a combination of equations (A 1) and (A 2) of Bahaj *et al.* (2007), used in their transfer of turbine test results in a confined flow to conclusions for unconfined conditions.

We first examine the limit of $\epsilon \ll 1$ for which we have the Lanchester–Betz results (2.12) and (2.13) for a turbine in free flow. Hence (2.19) gives

$$\frac{P}{P_{\text{ref}}} = \frac{u_0 + u_3}{2u_0}. \tag{2.24}$$

Thus the isolated turbine is most effective in comparison with a uniform tidal fence if u_3 is taken as u_0, but then the actual power generated goes to zero! If the power P from the turbine is maximized by taking $u_3 = \frac{1}{3}u_0$, then $P/P_{\text{ref}} = 2/3$ (Corten 2001). This is an encouraging result, showing that the loss of power by using an isolated turbine, or a tidal fence occupying only a small part of a channel cross-section, is only one-third of the power that could be generated by using the whole cross-section.

We next examine the other limit, with the turbine occupying nearly all of the channel cross-section so that $(1-\epsilon) \ll 1$. Then

$$u_4 \simeq 2(1-\epsilon)^{-1}(u_0 - u_3), \qquad u_1 \simeq u_3 \tag{2.25}$$

and

$$P = 2(1-\epsilon)^{-2} A u_3 (u_0 - u_3)^2. \tag{2.26}$$

This has a maximum when $u_3 = \frac{1}{3}u_0$ (interestingly the same as for an isolated turbine) of

$$P_{\max} = \tfrac{16}{27}(1-\epsilon)^{-2} \times \tfrac{1}{2}Au_0^3. \tag{2.27}$$

The extra 'efficiency' of $(1-\epsilon)^{-2}$ compared with the Lanchester–Betz limit for an isolated turbine occurs because the confining influence of the channel boundaries greatly increases the pressure drop that can occur.

Also in this limit

$$\frac{P}{P_{\text{ref}}} = \frac{u_3}{u_0}. \tag{2.28}$$

Thus, as for $\epsilon \ll 1$, the turbines are most effective in the limit of u_3 approaching u_0, but, again, this implies no power. If the power itself is maximized, with $u_3 = \frac{1}{3}u_0$, then $P/P_{\text{ref}} = 1/3$.

We have discussed the problem as if there were a single turbine in the channel, though the analysis would apply just as well for multiple turbines, assuming that the current speed is the same through each one. In this case, A represents the cross-sectional area of all the turbines combined.

So far we have shown that, if turbines occupying just a fraction of the cross-sectional area of a channel are optimized for local power generation, they tap 2/3 of the available power if the fraction is small, but only 1/3 if the fraction is large. We now consider intermediate cases.

2.1. *Solution for all values of A/A_c*

We use the solution of u_4 from (2.23) in (2.10) so that P may be expressed as $\frac{1}{2}Au_0^3$ times a function of u_3/u_0 only for any chosen value of ϵ. For each value of ϵ we find the value of u_3/u_0 which maximizes $P/(\frac{1}{2}Au_0^3)$.

Numerical solution shows that $P/(\frac{1}{2}Au_0^3)$ is maximized for $u_3/u_0 = 1/3$ for all values of ϵ. This is not obvious analytically, but may be confirmed by demonstrating that, when $\mathrm{d}P/\mathrm{d}u_3 = 0$ in order to find $P_{\max}$, the implied derivative $\mathrm{d}u_4/\mathrm{d}u_3$ when $u_3/u_0 = 1/3$ is the same as that given by the derivative of (2.22) with respect to u_3.

Proceeding with this general result that $u_3/u_0 = 1/3$, we find that

$$\frac{u_1}{u_0} = \frac{2}{3(1+\epsilon)}, \qquad \frac{u_4}{u_0} = \frac{3+\epsilon}{3(1-\epsilon)}. \tag{2.29}$$

$P_{\max}$ is given by (2.27) for $0 \leqslant \epsilon < 1$. We note that the factor $(16/27)(1-\epsilon)^{-2}$ multiplying the kinetic energy flux through area A in (2.27) exceeds 1 if $\epsilon > 0.23$. The ratio of $P_{\max}$ to the reference power is

$$\frac{P_{\max}}{P_{\text{ref}}} = \frac{2}{3(1+\epsilon)}. \tag{2.30}$$

These general formulae agree with the earlier results for the limiting cases of ϵ small or close to 1.

Other quantities of interest are

$$p_4 - p_0 = \frac{4\epsilon(3-\epsilon)}{9(1-\epsilon)^2}u_0^2, \qquad p_5 - p_4 = \frac{8\epsilon}{9(1-\epsilon)}u_0^2. \tag{2.31}$$

Also, if the drag F on the turbine is related to the upstream current speed u_0 by a drag coefficient C_D, with $F = C_D A u_0^2$, then at maximum power

$$C_D = \frac{4}{9}\frac{(1+\epsilon)}{(1-\epsilon)^2}, \tag{2.32}$$

which tends to 4/9 for small ϵ but becomes large as ϵ approaches 1.

It is remarkable that a problem which seems to display considerable algebraic complexity leads to such simple results for the key quantities. We have also confirmed that, for all values of ϵ, the ratio u_1/u_0, and hence P/P_{ref}, is a monotonically increasing function of u_3/u_0, approaching 1 as $u_3/u_0 \to 1$, at which point the actual power tends to zero.

At this point we may check one of the basic assumptions of our analysis, that the cross-sectional area of the channel is unchanged as the flow passes through the turbines and adjusts downstream. This is valid for a channel of a given shape only if the change in the height of the water surface is a small fraction of the water depth h. This change in surface elevation is given by the change in pressure divided by g (as mentioned earlier, we are omitting the water density for the sake of economy and clarity). The lowest free-surface level is at the section with pressure p_4, and the change in water level from upstream to there is small compared with h if

$$\frac{u_0^2}{gh} \ll \frac{9(1-\epsilon)^2}{4\epsilon(3-\epsilon)}. \tag{2.33}$$

Our earlier requirement of a small Froude number $u_0(gh)^{-1/2}$ is therefore generally adequate but, as ϵ increases, the requirement becomes more stringent. This is because the increasing value of u_4 is associated, by Bernoulli's theorem, with an increasing drop in water level.

2.2. *Exit control*

We have compared the maximum power with that available from a uniform tidal fence across the whole channel. This is a convenient comparison as the theory of GC05 provides the maximum value of P_{ref} for a channel, and the present theory now describes what fraction of this could be obtained by a partial fence, or fences, with the rest being lost to dissipation as streams merge.

This raises the question, though, of what happens if the streams do not merge. Is more power available if the dissipation associated with merging is avoided? To examine this we need to consider the dynamics of flow in a channel in conditions with lateral variation of the current.

A simple situation, and one of considerable applicability, is when the channel is short, with the dynamics controlled by exit separation rather than acceleration or friction in the channel itself. In this case, u_4 remains constant as it is always given by the Bernoulli equation as $(2g\zeta_0)^{1/2}$, where ζ_0 is the sea level difference from one end of the channel to the other, and we assume, as in GC05, that the channel connects two large basins in which the tidal elevation is unaffected by the presence of turbines in the channel. We now seek to maximize P from (2.10), regarding it as a function of u_3 with u_4 fixed and u_0 given in terms of u_3 and u_4 by (2.22).

For small ϵ, $u_0 \simeq u_4$, with the turbine hardly affecting the volume flux through the channel. The Lanchester–Betz limit applies with

$$P_{\max} = \tfrac{16}{27} \times \tfrac{1}{2} A u_4^3 = 0.59 \times \tfrac{1}{2} A u_4^3. \tag{2.34}$$

At the other limit, when $1-\epsilon$ is small, with the turbine(s) occupying most of the

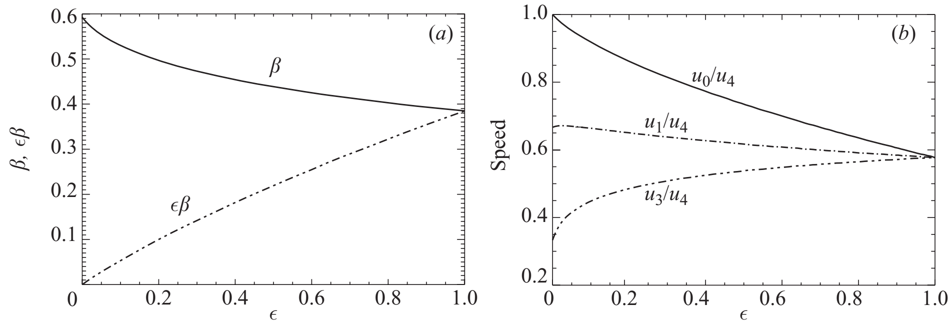

FIGURE 2. (a) The coefficients β and $\epsilon\beta$ from (2.37). (b) The speeds u_0, u_1 and u_3, all normalized by u_4.

channel cross-section, $u_0 \simeq u_3$ and

$$P = \tfrac{1}{2} A u_3 \left(u_4^2 - u_3^2\right). \tag{2.35}$$

This is essentially the result in GC05 for a complete tidal fence. We now have

$$P_{\max} = \frac{2}{3^{3/2}} \times \tfrac{1}{2} A u_4^3 = 0.38 \times \tfrac{1}{2} A u_4^3, \tag{2.36}$$

achieved when u_3/u_4 is $3^{-1/2} = 0.58$. Thus, if the maximum power is given by

$$P_{\max} = \beta \times \tfrac{1}{2} A u_4^3 = \epsilon\beta \times \tfrac{1}{2} A_c u_4^3, \tag{2.37}$$

then β decreases from 0.59 in (2.34) to 0.38 in (2.36) as the area occupied by the turbines increases from a very small fraction of the cross-section to most of it. Figure 2(a) shows β and $\epsilon\beta$ as functions of $\epsilon = A/A_c$. Note that, as β is a slowly decreasing fraction of ϵ, for a given A_c and u_4 the maximum power increases as ϵ increases, though it is slightly less than directly proportional. Figure 2(b) shows the normalized current speeds $u_0/u_4, u_1/u_4$ and u_3/u_4 as functions of ϵ; for small ϵ they are respectively 1, 2/3 and 1/3, but they all converge to $3^{-1/2} = 0.58$ as ϵ approaches 1.

3. Discussion

The first result of this paper has been that, for a given upstream velocity, the Lanchester–Betz maximum efficiency factor 16/27 for a turbine in an infinite medium is increased by a factor $(1 - A/A_c)^{-2}$ for a turbine of cross-sectional area A in a channel of cross-sectional area A_c. If it is assumed that a turbine is operating in the optimum mode, but that lateral mixing of the wake and free stream downstream of the turbine restores the flow to a uniform state, then some energy is dissipated in this mixing and the turbines are less effective than if they were deployed in a complete fence across the whole channel. The maximum energy produced by the turbine, or turbines, is a fraction of the potential of a complete tidal fence, with this fraction being 2/3 if the turbines occupy a small fraction of the channel cross-section and decreasing to 1/3 if they occupy most of the section.

The actual, rather than fractional, power still increases as the area ratio A/A_c increases. The optimum number N of partial fences along a channel, for a given area ratio $\epsilon = A/A_c$, may be obtained by combining the results here for P_{ref} with the maximum power for a channel derived by GC05. This would assume that the streams merge between each fence. Quantitatively, combining (2.27) and (2.30), we would have

$$N \times \frac{4\epsilon(1+\epsilon)}{9(1-\epsilon)^2} A_c u_0^3 = P_{GC} \tag{3.1}$$

where P_{GC} is given by the formula of GC05 discussed in the Introduction. A smaller number of fences could, of course, be used if it is desirable to reduce the decrease in the flow rate along the channel.

In a different scenario, we have assumed that the rapid flow which bypasses the turbine has a speed determined by flow separation at the channel exit. It is further assumed that this exit separation is the only factor limiting the flow, as is likely for a short channel. In this scenario the maximum power increases at a slightly slower rate than the area occupied by the turbines.

Another scenario would be for a quasi-steady basic state in which both bottom friction in the channel and exit separation are important, so that some combination of u_0^2 and u_4^2 remains constant as turbines are placed in the flow. We will not pursue this further here. There are also other interesting possibilities beyond the scope of the present paper. For example, in a long channel, one could consider putting a second turbine outside the wake of the first, taking advantage of the higher speed before any lateral mixing has occurred.

One general conclusion is that the problem of optimizing power generation by turbines in strong tidal flows benefits from consideration of idealized fluid dynamical situations.

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